# Computer-Aided Analysis and Design of Circular Waveguide Tapers

HEINRICH FLÜGEL AND EBERHARD KÜHN, MEMBER, IEEE

*Abstract* —Two different methods for the accurate numerical analysis of circular waveguide tapers are compared and applied to examine the usefulness of existing design methods. A modified Dolph–Chebychev taper turned out to give the best performance with respect to both minimum spurious mode excitation and taper length.

## I. Introduction

Circular waveguide tapers are used in gyrotrons to connect waveguides of different diameters. The requirements on such highly overmoded tapers are good match and spurious mode suppression at both waveguide ports. Up to now many different design procedures have been proposed ([1], [4]–[7]), but it is difficult to decide which of them is the most suitable under different design constraints. The authors have developed two different computer programs for the rigorous analysis of circular waveguide tapers. In this paper these methods of analysis are compared and applied to several tapers in order to verify and contrast the different design procedures.

## II. Taper Analysis

The first program to analyze a given taper integrates the coupled wave equations [1]

$$\frac{d}{dz}\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ I_1 \\ I_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & \cdots & -\gamma_1 Z_1 & 0 & \cdots \\ C_{21} & C_{22} & & 0 & -\gamma_2 Z_2 & \\ \vdots & & & \vdots & & \\ -\gamma_1/Z_1 & 0 & \cdots & -C_{11} & -C_{21} & \cdots \\ 0 & -\gamma_2/Z_2 & & -C_{12} & -C_{22} & \\ \vdots & & & \vdots & & \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ I_1 \\ I_2 \\ \vdots \end{bmatrix} \tag{1}$$

numerically, where $V$ and $I$ denote the complex amplitudes of the transverse electric and magnetic field vectors of the waveguide modes, $Z$ represents their characteristic impedance, and $\gamma$ is the propagation constant. $z$ is the coordinate in the direction of the taper axis (Fig. 1) and also the direction of wave propagation. The coupling coefficients are given by

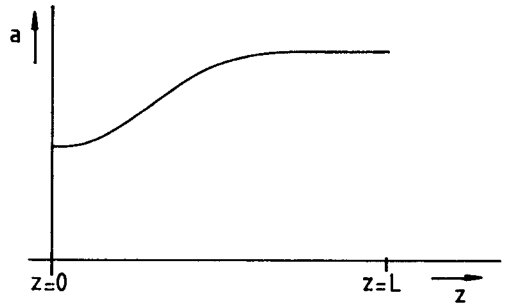



Fig. 1. Longitudinal section of a circular waveguide taper.

$$C_{np} = \iint_{(s)} \vec{e}_n \frac{d\vec{e}_p}{dz}\, dS \tag{2}$$

where $S$ represents the cross section of the waveguide and $\vec{e}$ the eigenvector of a mode with transverse electric field $\vec{E} = \vec{e}V$.

The integration of (1) leads to the hybrid matrix

$$\begin{bmatrix} V_b \\ I_b \end{bmatrix} = [K] \cdot \begin{bmatrix} V_a \\ I_a \end{bmatrix} \tag{3}$$

where the subscripts $a$ and $b$ denote the input and output ports. The differential equation for $[K]$ in (3) reads

$$\frac{d}{dz}[K] = [C] \cdot [K] \tag{4}$$

with the coupling matrix $[C]$ from (1). The starting solution for the numerical integration is $[K] = [1]$ (unit matrix). Due to numerical problems caused by evanescent modes, the taper is divided into several sections. The hybrid matrix of each section is calculated separately, and subsequently translated into a scattering matrix. The scattering

Manuscript received April 28, 1987; revised October 13, 1987.
H. Flügel is with the Technische Universität Hamburg-Harburg, Arbeitsbereich Hochfrequenztechnik, D-2100 Hamburg 90, West Germany.
E. Kühn is with the Forschungsinstitut der Deutschen Bundespost, D-6100 Darmstadt, West Germany.
IEEE Log Number 8718582.

0018-9480/88/0200-0332$01.00 ©1988 IEEE

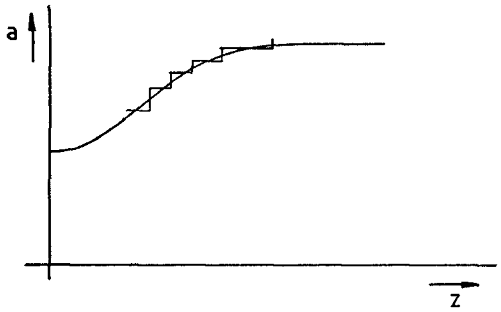



Fig. 2. Approximation of the taper by waveguide steps and uniform waveguide sections.

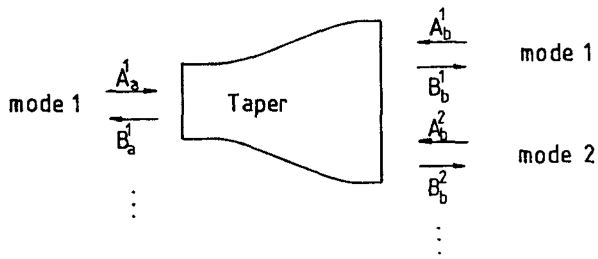



Fig. 3. Multimode scattering matrix of a taper.

matrices of all sections are linked together to form a resulting matrix which describes the electromagnetic property of the entire taper [2].

The second program makes use of a subdivision of the taper into waveguide steps and uniform waveguide sections, as shown in Fig. 2. The scattering matrix of a single step is calculated by a mode-matching method [3], and the scattering matrices of the individual waveguide junctions are cascaded to obtain the resulting scattering matrix

$$\begin{bmatrix} B_a \\ B_b \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} A_a \\ A_b \end{bmatrix} \tag{5}$$

of the taper. $A$ and $B$ denote the amplitudes of all waves propagating in the forward and backward directions, as depicted in Fig. 3.

To achieve high accuracy with this approximation, we have to take into account a certain number of evanescent modes. If we use sufficiently small steps, this program leads to the same results as the previous approach. The computation time required by the two programs is comparable.

## III. Comparison of Different Taper Design Procedures

The design of circular waveguide tapers for use in long-distance waveguide transmission systems has been studied by several authors [4]–[7] and a comprehensive summary can be found in [1]. In gyrotrons circular waveguide tapers are often used to connect the small resonator and the large output waveguide. The usual operational frequencies of gyrotrons are 70, 140, and 150 GHz. Typical gyrotron output tapers presently in use have lengths of 0.6 to 0.8 m. The design of tapers having more compact dimensions while maintaining the required levels of return loss and spurious mode suppression is a still unsolved problem.

In smooth transitions between waveguides of different diameters, we have very low reflections. So we can neglect the waves propagating in the backward direction. The coupled wave equations can then be written in the form

$$\frac{d}{dz}\begin{bmatrix} A_n \\ A_m \end{bmatrix} = \begin{bmatrix} -j\beta_n & \kappa_{nm} \\ \kappa_{mn} & -j\beta_m \end{bmatrix} \begin{bmatrix} A_n \\ A_m \end{bmatrix} \tag{6}$$

where $A$ denotes the amplitudes of waves propagating in the forward direction, $\beta$ is their phase constant, and $\kappa$ is the coupling factor. In [1], this equation is solved approximately, neglecting the reconversion of power from the unwanted mode $A_m$ to the wanted mode $A_n$. For modes above cutoff the design equations given below are evaluated in [1].

The magnitude of the unwanted mode is calculated from

$$|A_m| = W(\eta) = \int_{-\theta}^{\theta} CK(\xi)\exp(-j\eta\xi)\, d\xi. \tag{7}$$

The coupling distribution function $K(\xi)$ is defined by

$$CK(\xi) = \frac{j\kappa_{mn}}{1/\eta(\gamma_n - \gamma_m)} = k_0 a \frac{da}{dz} \frac{4x_m x_n}{(x_m^2 - x_n^2)^2} \tag{8}$$

with

$$\eta = \frac{f_0}{f} \geqslant 1. \tag{9}$$

Here $f$ is the operational frequency, $f_0$ is a certain upper limit for the frequency, and

$$k_0 = 2\pi f_0 \sqrt{\mu\epsilon}\,. \tag{10}$$

The quantity $x_n$ in (8) defines the cutoff wavenumber $x_n/a$ of mode $n$. The coupling distribution function has to be chosen such that, for a minimum value of $\theta$, the spurious mode amplitude $W(\eta)$ remains below the given limit $W_{\max}$ and the following equations are satisfied:

$$K(-\theta) = K(\theta) = 0 \tag{11}$$

$$\int_{-\theta}^{\theta} K(\xi)\, d\xi = 1. \tag{12}$$

The transition contour is calculated by numerical integration from

$$\ln\left(\frac{a(\xi)}{a_1}\right) = \ln\frac{a_2}{a_1}\int_{-\theta}^{\xi} K(\xi')\, d\xi' \tag{13}$$

$$z(\xi) = \frac{2k_0}{x_m^2 - x_n^2}\int_{-\theta}^{\xi} a^2(\xi')\, d\xi'. \tag{14}$$

Most of the known design procedures are based on (7)–(14). In [1] two different solutions for the function $K(\xi)$ are given. The first solution makes use of a Fourier series representation

$$K(\xi) = \sum_{i=1,3,5\cdots}^{N} a_i \cos\left(i\frac{\pi\xi}{2\theta}\right). \tag{15}$$

A rule is outlined in [1] to calculate the coefficients $a_i$ numerically. To arrive at compact transitions, we have to

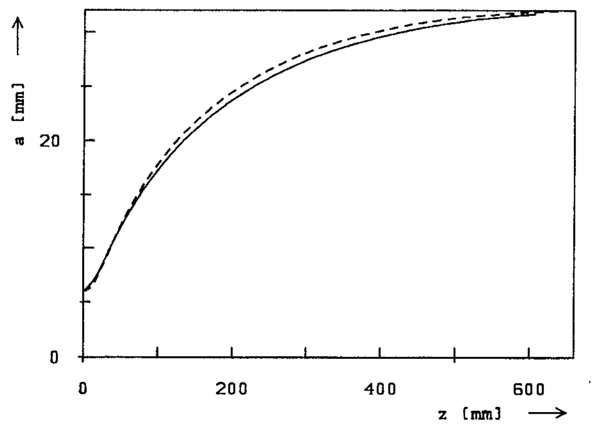



Fig. 4. Contour of tapers. (a) Dolph–Chebychev taper with end discontinuities (solid line). (b) Modified Dolph–Chebychev taper (dashed line).

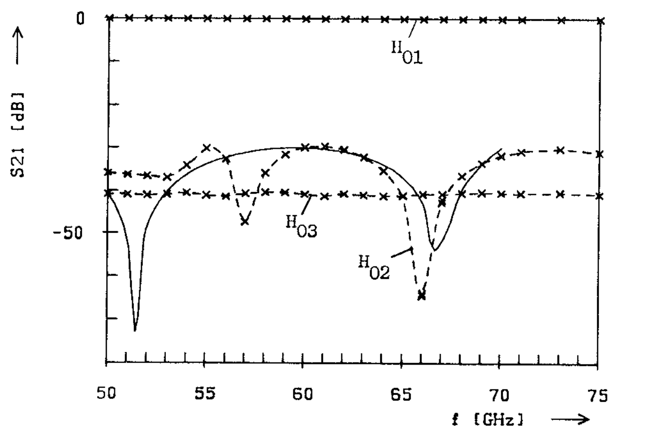



Fig. 5. Level of unwanted moded versus frequency (Dolph–Chebychev taper). (a) Theoretical level of $H_{02}$ mode (solid line). (b) Numerical analysis (dashed lines).

choose $N$ very large. For $N \to \infty$ the Dolph–Chebychev distribution is obtained:

$$K(\xi) = \frac{D}{2}\left\{\frac{I_1\left[\theta\sqrt{1-(\xi/\theta)^2}\right]}{\sqrt{1-(\xi/\theta)^2}} + \delta(\xi-\theta) + \delta(\xi+\theta)\right\} \tag{16}$$

where $I_1$ denotes the modified Bessel function of first order and $\delta(t)$ is Dirac's delta function.

In the latter case, the level of the unwanted mode follows from

$$W(\eta) = C \cdot \frac{\cos\left[\theta\sqrt{\eta^2-1}\right]}{\cosh\theta}. \tag{17}$$

This solution of $K(\xi)$ results in the shortest taper for a given upper limit $f_0$ and given level of the unwanted mode. The delta functions in $K(\xi)$, however, would lead to steps at both ends of the taper, and these steps give rise to the generation of higher order modes. This effect is not included in the design procedure, so that the performance of the taper may change drastically. As an example, we have designed a taper for this "optimum" coupling distribution function subject to the following specifications:

| | | |
|---|---|---|
| wanted mode $H_{01}$ | $f_0 = 70$ GHz | $W_{max} = -30$ dB |
| $a_1 = 6$ mm | $a_2 = 32$ mm | |
| $D = 0.012$ | $C = 2.606$ | $\theta = 5.105$. |

The resulting length of the taper amounts to 605.6 mm. Fig. 4 (curve (a)) shows the taper profile , and Fig. 5 gives the level of the unwanted $H_{02}$ mode according to (17). With the computer program which approximates the taper contour in step-ladder fashion (about 1000 steps), we are able to accurately analyze this taper. The result is also depicted in Fig. 5 and reveals that the design procedure estimates the maximum mode conversion level quite accurately.

In [8] a modified Dolph–Chebyshev distribution is used which avoids discontinuities at the taper ends:

$$K(\xi) = \frac{D}{2}\frac{I_0\left[\theta\sqrt{1-(\xi/\theta)^2}\right]}{\sinh\theta} \tag{18}$$

where $I_0$ is the modified Bessel function of zero order. The Fourier transform (7) leads to

$$W(\eta) = C\frac{\theta}{\sinh\theta}\frac{\sin\left[\theta\sqrt{\eta^2-1}\right]}{\theta\sqrt{\eta^2-1}}. \tag{19}$$

The first maximum of $W(\eta)$ is

$$D = C \cdot \frac{\theta}{\sin\theta} \cdot 0.21723. \tag{20}$$

Making use of the coupling distribution function $K(\xi)$, we arrive at

wanted mode $H_{01}$ $\quad f_0 = 63$ GHz $\quad W_{max} = -30$ dB

at a taper with

$\theta = 5.233 \quad L = 656.9$ mm.

Its contour is depicted in Fig. 4 as curve (b). Fig. 6 gives the theoretical level of the unwanted $H_{02}$ mode. This taper has no discontinuities and hence we are able to predict its performance with both computer codes in good agreement. The result of the numerical analysis is also depicted in Fig. 6 and the performance of this taper is obviously better than that of the "optimum" design. The pronounced discrepancy between the design objective and the actual characteristic originates from the simplifications involved in the design procedure, mainly from disregarding the reconversion of power from the unwanted mode into the wanted mode.

A solution which does not apply this assumption can be found in [1] and [9] using quasi-diagonalization of the coupled wave equations. In this approach the influence of higher order spurious modes is included as well. The level

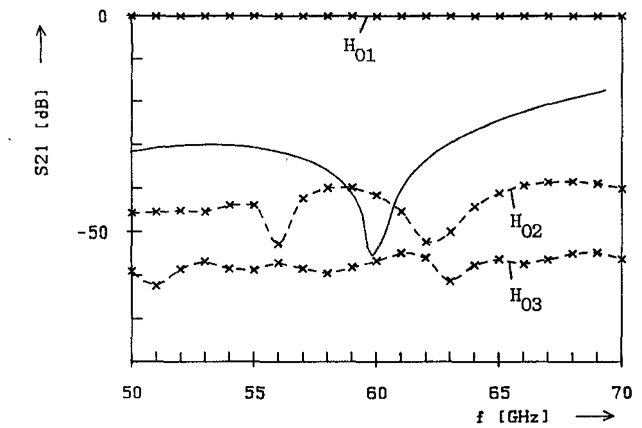



Fig. 6. Level of unwanted modes versus frequency (modified Dolph–Chebychev taper). (a) Theoretical level of $H_{02}$ mode (solid line). (b) Numerical analysis (dashed lines).

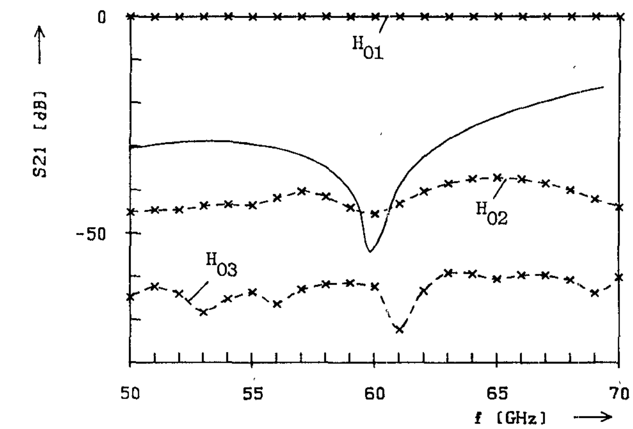



Fig. 7. Level of unwanted modes of the taper of improved design. (a) Theoretical level of $H_{02}$ mode (solid line). (b) Numerical analysis (dashed lines).

of the unwanted mode $m$ is calculated as

$$W(\eta)=\frac{1}{\eta}\int_{-\theta}^{\theta}\left[C\frac{dK}{d\xi}-j\alpha_1 C^2K^2 + j\alpha_2 C^3 K\int_{\xi}^{\theta}KI^2\,d\xi'\right]\exp(j\eta\xi)\,d\xi \quad (21)$$

where

$$\alpha_1=\sum_{n\neq l\neq m}^{l\leqslant p}\frac{(x_m^2-x_n^2)^3\left[x_l^2-(x_m^2+x_n^2)/2\right]x_l^2}{(x_l^2-x_n^2)^2(x_l^2-x_n^2)^2x_nx_m} \quad (22)$$

and $\alpha_2=0$ for circular waveguides. $p$ is the number of the highest-order mode which is taken into account. The coupling distribution $K(\xi)$ can be calculated from a suitably selected effective distribution $K_0(\xi)$ by numerically integrating

$$C\frac{dK}{d\xi}-\alpha_1C^2K^2-\alpha_2C^3K\int_{-\theta}^{\xi}K^2\,d\xi'=C_1\frac{dK_0}{d\xi}+C_2K_0 \quad (23)$$

with the starting solution $K(-\theta)=0$. $C_1$ and $C_2$ must be determined such that

$$K(\theta)=0 \quad \text{and} \quad \int_{-\theta}^{\theta}K(\xi)\,d\xi=1 \quad (24)$$

hold. $K_0$ was chosen to be the modified Dolph–Chebychev distribution (18). The taper contour can then be determined from (13) and (14) with $K(\xi)$ according to (23). With

wanted mode $H_{01}$ $f_0=63$ GHz $W_{max}=-30$ dB

$a_1=6$ mm $a_2=32$ mm

we have obtained

$C=2.606$ $D=0.012$

$\theta=5.233$ $\alpha_1=0.642$

$C_1=2.931$ $C_2=-0.591$

$L=634.87$ mm.

The resulting taper contour differs only slightly from that of Fig. 4(b). Fig. 7 depicts the theoretical level of the $H_{02}$ mode and the results from the numerical analysis. This taper shows comparable performance, but it is shorter than the modified Dolph–Chebychev design.

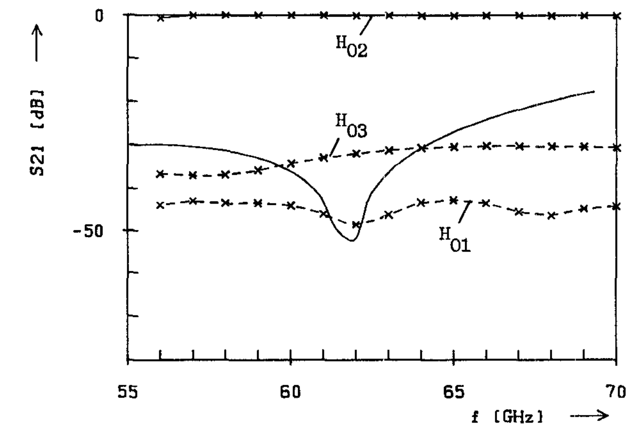



Fig. 8. Level of unwanted modes of modified Dolph–Chebychev taper with $H_{02}$ mode excitation. (a) Theoretical level of $H_{03}$ mode (solid line). (b) Numerical analysis (dashed lines).

The design based on (21) assumes the unwanted mode to be far above cutoff. This assumption does often not apply to tapers used in gyrotrons. To examine whether this design can also be applied slightly below or near cutoff, we have designed modified Dolph–Chebychev tapers using (7) (reconversion neglected) and (21) (reconversion considered) and assuming an excitation by the $H_{02}$ mode:

$f_0=64$ GHz $a_1=6$ mm $a_2=32$ mm

$W_{max}=-30$ dB ($H_{03}$ mode!).

Design with (7):

$D=0.00718$ $C=4.402$

$\theta=5.872$ $L=478.12$ mm.

Design with (21):

$D=0.00718$ $C=4.402$

$\theta=5.872$ $\alpha_1=0.38$

$C_1=4.828$ $C_2=-0.9215$

$L=460.97$ mm.

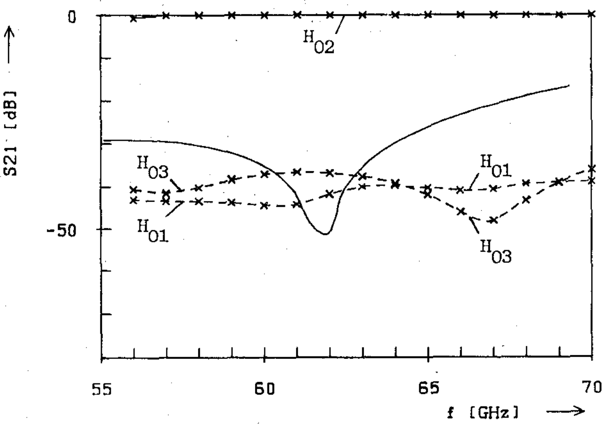



Fig. 9. Modified Dolph–Chebychev taper of improved design with $H_{02}$ mode excitation. (a) Theoretical level of $H_{03}$ mode (solid line). (b) Numerical analysis (dashed lines).

In Figs. 8 and 9, the amplitude levels of the individual modes are plotted versus frequency. The diagrams demonstrate that the performance is substantially better than specified. The design based on (21) is superior to that obtained from (7) with respect to both the spurious mode excitation and the resulting taper length. The disagreement between design and analysis is caused by (i) the violation of the assumption that the main spurious mode is far above cutoff, and (ii) the buildup of a finite return loss of the $H_{02}$ mode, which is entirely neglected in the design process.

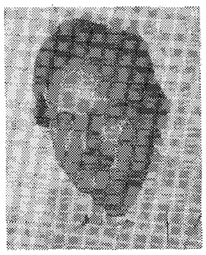
## IV. Conclusions

Two computer programs have been developed for the rigorous performance analysis of circular waveguide tapers. They are based on the direct integration of the coupled wave equations and on a mode-matching procedure applied to a step-ladder equivalent of the taper, respectively. Close agreement has been obtained between the predictions of these two approaches.

The computer programs are utilized to examine the usefulness of existing taper design procedures. It has been found that a Dolph–Chebychev design is actually capable of predicting the level of the first unwanted mode provided that the operational frequency is far above its cutoff. The spurious mode excitation can still be improved by taking into account the reconversion of power from the unwanted into the wanted modes. It has been verified by the computer analysis that the above design procedures are applicable even for frequencies near cutoff of the dominant spurious mode; however, that is at the expense of a certain degradation of the input reflection of the incident mode. A further improvement of the performance is possible only by direct computer optimization of the taper taking advantage of the analysis programs. But this approach should only be followed in the case of extremely unfavorable specifications since it necessitates excessively long computation times.

## References

[1] F. Sporleder and H.-G. Unger, *Waveguide Tapers, Transitions and Couplers*. London: Peter Peregrinus Ltd., 1979.
[2] H. Flügel, "Taper analysis of the 150-GHz KFK-gyrotron including the electron beam," *Int. J. Electron.*, vol. 64, pp. 95–106, 1988.
[3] E. Kühn and V. Hombach, "Computer-aided analysis of corrugated horns with axial or ring-loaded radial slots," in *Proc. ICAP 83*, part 1, 1983, pp. 127–131.
[4] C. C. H. Tang, "Optimization of waveguide tapers capable of multimode propagation," *Trans. Inst. Radio Engrs.*, vol. MTT-9, pp. 442–452, 1961.
[5] H.-G. Unger, "Circular waveguide taper of improved design," *Bell Syst. Tech. J.*, vol. 37, pp. 899–912, 1958.
[6] R. P. Hecken and A. Anuff, "On the optimum design of tapered waveguide transitions," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 374–380, 1973.
[7] W. Thumm, "Advanced overmoded waveguide components for high-power millimetre wave applications at 28, 70 and 140 GHz," in *Proc. 15th European Microwave Conf.* (Paris), 1985, pp. 790–795.
[8] R. P. Hecken, "A near-optimum matching section without discontinuities," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 734–739, 1972.
[9] F. Sporleder, "Waveguide transition design of improved accuracy," *Arch. Elek. Übertragung.*, vol. 30, pp. 289–296, 1976.



**Heinrich Flügel** was born in Bremen, Federal Republic of Germany, in 1958. He received the Dipl.-Ing. degree in electrical engineering from the Technical University of Braunschweig, Federal Republic of Germany, in 1983.

After one year of working in the electronics industry, he joined the Arbeitsbereich Hochfrequenztechnik at the Technical University of Hamburg-Harburg, where he is concerned with investigations on oversized waveguide systems.

**Eberhard Kühn** (M'85) was born in Berlin, Germany, on December 23, 1938. He received the Dipl.-Ing. and Dr.-Ing. degrees in electrical engineering from the Technical University of Braunschweig, West Germany, in 1966 and 1972, respectively.

In 1972, he joined the Research Institute of the Deutsche Bundespost, where he was involved in the design, optimization, and implementation of frequency multiplexers and 4-PSK modems for the German project on millimeter-wave transmission in circular waveguide. Since 1977, he has been Head of the Antenna and Wave Propagation Division of that Institute. His personal research activities are primarily devoted to electromagnetic theory, currently with emphasis on computed-aided modeling of microwave components (waveguide and resonator structures, corrugated horns, polarizers, filters) and on reflector antenna analysis.

Dr. Kühn is member of the German Society for Information Technology (ITG, formerly NTG). He received the NTG Award in 1975, and the Microwave Prize (with two coauthors) of the 14th European Microwave Conference, Brussels, 1984. Since 1976, he has been an Associate Editor of *NTZ-Archiv*, the scientific journal of the ITG.